**Remarks on Ann’s Reasonability Checks**

George Ho, 6/5/2016

Definition:

For brevity’s sake, the following two equations (but the second form in particular) will be referred to as *Andre’s equation*.

Realness:

1. There are solutions to an -degree polynomial, accounting for multiplicity. Thus, we can expect solutions to Andre’s equation, some of which may be complex.
2. We can trivially discard the complex solutions, as it does not make sense to have a complex average of real numbers, nor does it make sense to have a complex number as an annualized value.

Boundedness:

1. There is, in general, no simple relationship between the values of and the values of . In particular, there is no easy guarantee that the values of will be bounded by the maximum and minimum values of , and , respectively.
2. Furthermore, in light of the fact that Andre’s equation is to be solve for an average value of , it would make sense that this average value cannot be greater than or less than .
3. Therefore, it is worthwhile to check that all solutions are on the open interval , and discard any solutions that do not lie on this interval.

Monotonicity:

1. Consider the RHS of Andre’s equation:
2. Note two properties of RHS:
   1. Non-negativity: since return factors can never be negative, and since RHS is a product of such return factors (the portfolio return factors, to be exact), RHS itself cannot be negative.
   2. Monotonicity: by the same reasoning, an increase in *any one* of will result in an increase in RHS.
3. Now consider the graph with the value of RHS on the y-axis, and the value of A on the x-axis. Such a graph is a -degree polynomial, with a lead coefficient of 1. By setting RHS to equal a specific numerical value, Ann essentially draws a horizontal line and finds the intersection point(s) of the polynomial graph and the line.
4. If the line is to be shifted upwards (by increasing *any* , by property 2.b.), we can see that around half of the intersection values of A will increase, and around half will decrease.
5. To be specific:
   1. If is even, intersection values of A will increase, and intersection values will decrease.
   2. If is odd, intersection values of A will increase, and intersection values of A will decrease.
6. Mutatis mutandis if RHS decreases.
7. A more rigorous proof of this behavior would involve considering the derivative of the -degree polynomial (in particular, the sign of the derivative) and repeated applying Rolle’s Theorem.
8. Ultimately, we conclude that checking for monotonicity will eliminate “around half” of all possible real values of A.
9. An alternative way to implement this idea is to simply compute the slope at each intersection points: we only desire those intersection points with positive slopes (since they will increase as RHS increases, and decrease as RHS decreases).
10. This implementation avoids a subtle error with the method outlined in 4). If the horizontal line were to be increased or decreased by too much, it is possible that Ann will simply lose two solutions altogether, as the line will have moved above or below a vertex of the polynomial graph. This problem is not unfixable, but it would require some convoluted code: Ann would have to check to see if any solutions had been lost, and if so, adjust the increment/decrement of RHS accordingly. Computing the slope is a more elegant and effective solution.

Distance to the Naïve Difference of Annualizations:

1. The above three checks all eliminate the impossible or unreasonable solutions. The last thing to do would therefore be to pick “the best reasonable solution”.
2. This is done by picking the solution that is closest to the naïve difference in annualizations.
3. In other words, while , they are relatively close. ***But why? Explanation needed here.***
4. So we can use the naïve difference to estimate the actual annualized value of , and select the solution that is closest to the naïve difference.